YEAR 7
KNOWLEDGE ORGANISERS

BLOCK: ALGEBRAIC THINKING

Sequences
Algebraic notation
Equality and Equivalence

"MATHS OPENS DOORS"

YEAR 7 – ALGEBRAIC THINKING... Sequences

What do I need to be able to do?

By the end of this unit you should be able to:

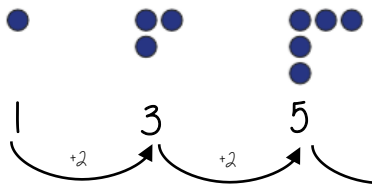
- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence

Keywords

Arithmetic: an arithmetic sequence is one which is made by adding the same amount each time.
Difference: the gap between two numbers, found by doing a subtraction
Geometric: a geometric sequence is one which is made by multiplying by the same amount each time
Linear: a sequence is linear when the gap between the terms is the same each time.
Non-linear: a sequence is non-linear when the gap between the terms is not the same each time
Position: where a term is in the sequence, e.g. 10th term is the 10th number/shape along
Rule: a way of explaining how to get the terms of a sequence
Sequence: a set of numbers, shapes or patterns in a particular order
Term: one of the numbers, letters, shapes or patterns in a sequence, series or algebraic expression

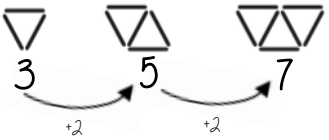
Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image



What will the next number be? Can you draw this?

Predict and check terms



CHECK – draw the next terms

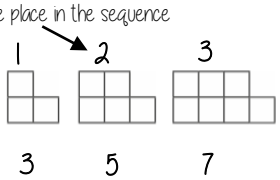
9 11 13

Predictions:
Look at your pattern and consider how it will increase.

e.g. How many lines in pattern 6?
Prediction - 13
If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

Position: the place in the sequence



3 5 7

"The **term** in **position** 3 has 7 squares"

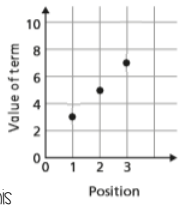
Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.

Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?
It increases by adding 4 to each term

How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence

Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?
It increases by multiplying the previous term by 2 – this is a geometric sequence because the constant is multiply by 2

How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence

Explain term-to-term rule

How you get from term to term
Try to explain this in full sentences not just with mathematical notation
Use key maths language – doubles, halves, multiply by two, add four to the previous term etc.
To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term. The sequence begins at 4.

4, 12, 36, 108...
x3 x3 x3
First term

YEAR 7 – ALGEBRAIC THINKING...

Algebraic notation

What do I need to be able to do?

By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions.
- Represent functions graphically.

Keywords

Commutative: a mathematical process is commutative if the numbers may be inputted in any order
Evaluate: to find a numerical value for an expression, to 'work it out'
Expression: one or a group of numbers, variables and mathematical operations representing a number or quantity
Function: a mathematical relationship between two values
Input: what is put into a function
Inverse (operation): the opposite or reverse operation
Linear: a sequence is linear when the gap between the terms is the same each time
Operation: a mathematical process such as addition, subtraction, multiplication, division, squaring, square rooting, etc.
Output: what comes out of a function
Sequence: a set of numbers, shapes or patterns in a particular order
Substitute: to put numerical values in place of the letters in an expression

Single function machines

INPUT → [ ] → OUTPUT
The number that goes IN — The number that comes out

This box gives the calculation instruction

To find the input from the output
Use the **INVERSE** operation

Using letters to represent numbers

5 + 5 + 5	y + y + y + y	20 ÷ h
3 x 5	y x 4	$\frac{20}{h}$
5 x 3	4 x y	
	4y	

Addition and multiplication can be done in any order **Commutative calculations**

4 lots of 'y'

20 shared into 'h' number of groups

Single function machines (algebra)

INPUT → [x 10] → OUTPUT

a → 10a
3c → 30c

÷ 10

To find the input from the output
Use the **INVERSE** operation

Find functions from expressions

INPUT 7x → [?] → OUTPUT 14x

Find the relationship between the input and the output

Sometimes there can be a number of possible functions.
e.g. **+7x** or **x 2** could both be solutions to the above function machine

Substitution into expressions

4y ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4 x 7 **OR** 7 + 7 + 7 + 7 **OR** 7 x 4 = 28

e.g. $y - 2$
$= 7 - 2 = 5$

Two step function machines

INPUT → [ ] → [ ] → OUTPUT

Calculate the value at the end of each operation

For the input use the **INVERSE** operations

Two step function machines (algebra)

b → [x 5] → 5b → [+ 4] → 5b + 4
[÷ 5] [- 4]

IMPORTANT
Calculate the value at the end of each operation

c → [+ 4] → c+4 → [x 5] → 5(c + 4) = 5c + 20
[- 4] [÷ 5]

NOTE:
The whole first output is multiplied by 5

Find functions from expressions

f → [x 5] → [+ 3] → $\frac{f + 5}{3}$

f add 5 then divide by 3

f → [÷ 3] → [+ 5] → $\frac{f}{3} + 5$

f divided by then add 5

NOTE: the difference in the two expressions

Sometimes it helps to try to explain the expression in word – and consider what has happened to the input

Substitution into an expression

$2(x + 3)$

Add 3 to the input then times 2

Put the expression into a function machine

INPUT → [+ 3] → [x 2] → OUTPUT

If x = 10
10 + 3 = 13.... 13 x 2 = 26

Forming a sequence

INPUT	1	2	3
OUTPUT	8	10	12

$2(x + 3)$

The substitution is the 'input' value
The OUTPUT becomes the sequence

Representing functions graphically

Take the function and generate a sequence

INPUT → [+ 3] → [x 2] → OUTPUT **2(x + 3)**

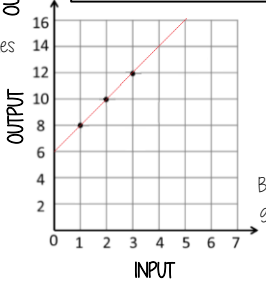
To represent graphically the input becomes x co-ordinates and the output becomes y co-ordinates

$y=2(x + 3)$

INPUT (x)	1	2	3
OUTPUT (y)	8	10	12

This becomes a co-ordinate pair (2, 10) to plot on a graph

Not all graphs will be linear only those with an integer value for x.
Powers and fractions generate differently shaped graphs.



NOTE:
Because this is a linear graph you can predict other values

YEAR 7 – ALGEBRAIC THINKING...

Equality and Equivalence

What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

Keywords

Coefficient: a number which multiples a variable
Equality: the state of being equal
Equal: having the same amount of value
Equation: a mathematical statement connecting two things which are equal. It will contain an equals sing '='.
Expression: one or a group of numbers, variables and mathematical operations representing a number or quantity
Index (Indices is plural): a small number written to the upper right of a number or variable which shows how many times the number or letter is multiplied by itself
Inverse (operation): the opposite or reverse operation
Like terms: terms whose variables (such as x and y) and indices (such as the 2 in x^2) are the same
Solution: the answer to a problem
Solve: a command word: find the unknown letter(s) in this problem
Term: one of the numbers, letters, shapes or patterns in a sequence, series of algebraic expression

Equality

$2 + 14 = 5 + 5 + 6$

16 16

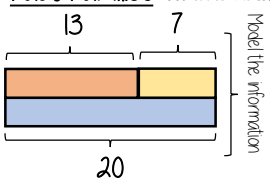
"Is equal to"

The sum on the left has the same result as the sum on the right

Saying it out loud sometimes helps you to understand equality

Fact Families

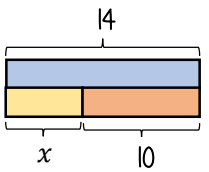
Use a bar model to display the relationships between terms and numbers



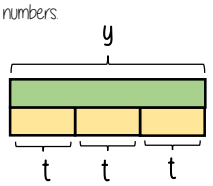
Model the information

Fact Family

$13 + 7 = 20$ $20 - 7 = 13$
$7 + 13 = 20$ $20 - 13 = 7$



$x + 10 = 14$ $14 - 10 = x$
$10 + x = 14$ $14 - x = 10$



$t + t + t = y$ $y - t - t = t$
$3 \times t = y$ $y \div 3 = t$
$3t = y$ $y \div t = 3$

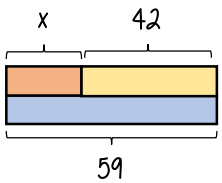
Solve one step equations (+/-)

There is more to this than just spotting the answer

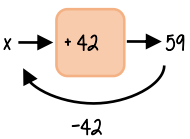
$x + 42 = 59$

$x + 42 = 59$
$42 + x = 59$

$59 - x = 42$
$59 - 42 = x$



Don't forget you know how to use function machines

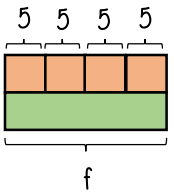


Solve one step equations (x/÷)

$\frac{f}{4} = 5$

$f \div 4 = 5$
$f \div 5 = 4$

$5 \times 4 = f$
$4 \times 5 = f$



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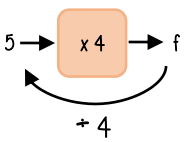


Like and unlike terms

Like terms are those whose variables are he same

♡ and 3♡ are **like** terms

the variable is the same

★ 3♡ are **unlike** terms

the variables are NOT the same

Examples and non-examples

Like terms	Un-like terms
y, 7y	y, 7x
$2x^2$, x^2	$2x^2$, $2c^2$
ab, 10ba	ab, 10a
5, -2	5, -2t

Note here ab and ba are commutative operations, so are still like terms

Equivalence

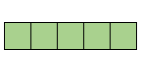
Check equivalence by substitution
e.g. m= 10

5m	2 x 2m	7m – 3m
5 x 10	2 x (2x 10)	(7x 10) – (3x 10)
= 50	= 2 x 20	= 70 – 30
	= 40	= 40

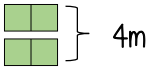
Equivalent expressions

Repeat this with various values for m to check

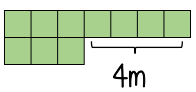
5m



2 x 2m



7m – 3m



Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms

Only **like terms** can be combined

$4x + 5b - 2x + 10b$

$4x + 5b - 2x + 10b$

$2x + 15b$

Common misconceptions

$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x2 and x terms are un-like terms so can not be collected