



YEAR 7
KNOWLEDGE ORGANISERS



BLOCK: REASONING WITH NUMBER

Developing number sense

Sets and probability

Prime numbers and proof

"MATHS OPENS DOORS"

YEAR 7 — REASONING WITH NUMBER

Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

- Associative:** in addition or multiplication, no matter how the numbers are grouped together the answer will always be the same. Subtraction and division are not associative.
- Commutative:** a mathematical process is commutative if the numbers may be inputted in any order.
- Dividend:** the amount that you want to divide up: $\text{dividend} \div \text{divisor} = \text{quotient}$
- Divisor:** the number we divide by: $\text{dividend} \div \text{divisor} = \text{quotient}$
- Equation:** a mathematical statement connecting two things which are equal. It will contain an equals sign "="
- Expression:** one or a group of numbers, variables and mathematical operations representing a number or quantity
- Quotient:** the result when one number is divided by another: $\text{dividend} \div \text{divisor} = \text{quotient}$

Mental methods for addition/ subtraction

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help $4000 \div 25$
"How many 25's in 100" then how many chunks of that in 4000.

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller e.g. $0.1 = \div 10$

Methods for multiplication 12×0.03

$$\begin{array}{l} 12 \times 3 = 36 \\ 12 \times 3 = 36 \\ 12 \times 0.3 = 3.6 \\ 12 \times 0.03 = 0.36 \end{array} \quad \begin{array}{l} 12 \times 3 = 36 \\ +10 \downarrow +100 \downarrow +1000 \downarrow \\ 12 \times 0.03 = 0.36 \end{array}$$

Methods for addition $2.3 + 2.4$

$$\begin{array}{l} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

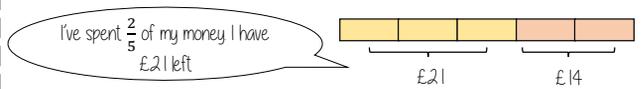
Methods for division $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

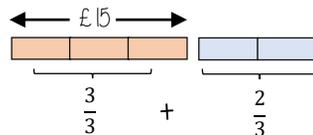
$$\begin{array}{l} 1.5 \div 0.05 \\ \times 100 \downarrow \quad \times 100 \downarrow \\ 150 \div 5 = 30 \end{array}$$

Mental methods for fractions

Use bar models where possible



How much did they have to begin with?



What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach $300 + 900$.

The correct estimation would be $200 + 900 = 1100$.

Number facts

Use $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 50$$

For division you must consider the impact of the divisor becoming smaller or bigger.
Smaller — the answer will be bigger (It is being shared into less parts)
Bigger — the answer will be smaller (It is being shared into more parts)

Algebraic facts

$$2a + 2b = 10 \quad \text{Everything } \times 2$$

$$0.1a + 0.1b = 0.5$$

Everything $\div 10$

$$a + b = 5$$

Add 2 to the total

$$a + b + 2 = 7$$

The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 7 — REASONING WITH NUMBER

Sets and probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

- Bias:** a systematic, built-in error which makes all values wrong, or the outcome of an experiment loaded towards a particular outcome
- Element:** a member of a set
- Fair:** free from bias
- Intersection:** the place where two lines cross one another or where two sets overlap
- Mutually exclusive:** events that cannot happen at the same time in a probability experiment
- Probability:** how likely it is that an event will occur
- Random:** a chance pick, like drawing a number out of a hat, in which each object has an equal chance of being selected
- Set:** a collection of "things" (objects or numbers)
- Union:** the set made by combining the elements of two sets

Identify and represent sets

The **universal set** has this symbol ξ — this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things — you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$

$A = \{1, 4, 9, 16, 25, 36, 49\}$

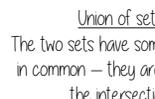
All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams



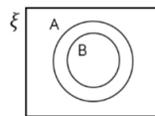
Mutually exclusive sets

The two sets have nothing in common
No overlap



Union of sets

The two sets have some elements in common — they are placed in the intersection



Subset

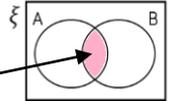
All of set B is also in Set A so the ellipse fits inside the set

The box

Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

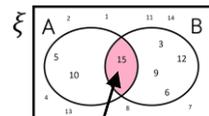
Intersection of sets

Elements in the intersection are in set A AND set B



The notation for this is $A \cap B$

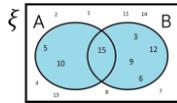
$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$



The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets



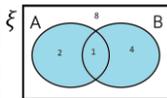
Elements in the union could be in set A OR set B

The notation for this is $A \cup B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15



This Venn shows the **number of elements** in each set

Sample space — for single events



A sample space for rolling a six-sided dice is $S = \{1, 2, 3, 4, 5, 6\}$



A sample space for this spinner is $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event



Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

$$P(\text{Blue}) = \frac{4}{10}$$

← There are 4 blue sectors
← There are 10 sectors overall

$$= \frac{2}{5}$$

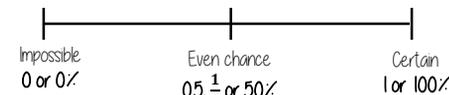
Probability notation
 $P(\text{event})$

Probability can be a fraction, decimal or percentage value

$$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$$

Probability is always a value between 0 and 1

The probability scale



The more likely an event the further up the probability it will be in comparison to another event
(It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$
∴ The probability of **NOT** getting a blue ball is $\frac{4}{5}$
The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



YEAR 7 — REASONING WITH NUMBER

Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

- Conjecture:** a statement that might be true (based on some research or reasoning) but it is not proven
- Counterexample:** an example which shows that a particular statement is false
- Expression:** one or a group of numbers, variables and mathematical operations representing a number or quantity
- Factor:** a whole number or variable that divides exactly into another number or expression
- Highest Common Factor (HCF):** the largest of all the common factors shared by two (or more) numbers.
- Lowest Common Multiple (LCM):** the smallest whole number that is a multiple of two (or more) numbers
- Multiple:** the result of multiplying a number by an integer
- Prime number:** an integer greater than 1 which has exactly two factors – itself and 1

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

3x, 6x, 9x ...

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3 x 15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

Factors of 10: 1, 2, 5, 10

5 x 2 or 2 x 5

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

Factors of 6x: 6, x, 1, 6x, 2x, 3, 3x, 2

6x x 1 OR 6 x x

2x x 3

3x x 2

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number

The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

odd, even, odd

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers): 1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Comparing fractions

Compare fractions using a LCM denominator

$\frac{3}{5}$ and $\frac{7}{10}$

$\frac{6}{10}$ and $\frac{7}{10}$

Product of prime factors

Multiplication part-whole models

30 = 2 x 15 = 2 x 3 x 5

30 = 3 x 10 = 3 x 2 x 5

30 = 5 x 6 = 5 x 2 x 3

All three prime factor trees represent the same decomposition

Multiplication is commutative

30 = 2 x 3 x 5

Multiplication of prime factors

Using prime factors for predictions

e.g. 60: 30 x 2 = 2 x 3 x 5 x 2

150: 30 x 5 = 2 x 3 x 5 x 5

Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only one counterexample is needed to disprove a conjecture