YEAR 8
KNOWLEDGE ORGANISERS

BLOCK: REASONING WITH NUMBER

Developing number sense
Prime numbers and proof

"MATHS OPENS DOORS"

YEAR 8 — REASONING WITH NUMBER

Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

Associative: in addition or multiplication, no matter how the numbers are grouped together the answer will always be the same. Subtraction and division are not associative.
Commutative: a mathematical process is commutative if the numbers may be inputted in any order.
Dividend: the amount that you want to divide up: dividend÷divisor=quotient
Divisor: the number we divide by: dividend÷divisor=quotient
Equation: a mathematical statement connecting two things which are equal. It will contain an equals sign "="
Expression: one or a group of numbers, variables and mathematical operations representing a number or quantity
Quotient: the result when one number is divided by another : dividend÷divisor=quotient

Mental methods for addition/ subtraction

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative

2 × 4 = 4 × 2

The order of multiplication does not change the result

Partitioning can help multiplication

24 x 6 = 20 x 6 + 4 x 6
= 120 + 24
= 144

Division is not associative

Chunking the division can help 4000 ÷ 25
"How many 25's in 100" then how many chunks of that in 4000.

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller e.g x 0.1 = ÷ 10

Methods for multiplication 1.2 x 0.03

12 × 3 = 36
1.2 × 3 = 3.6
1.2 × 0.3 = 0.36
1.2 × 0.03 = 0.036

12 × 3 = 36
÷10 ÷100 ÷1000
1.2 × 0.03 = 0.036

Methods for addition 2.3+2.4

2 + 2 = 4
0.3 + 0.4 = 0.7
4 + 0.7 = 4.7

Methods for division 1.5 ÷ 0.05

Multiply by powers of 10 until the divisor becomes an integer

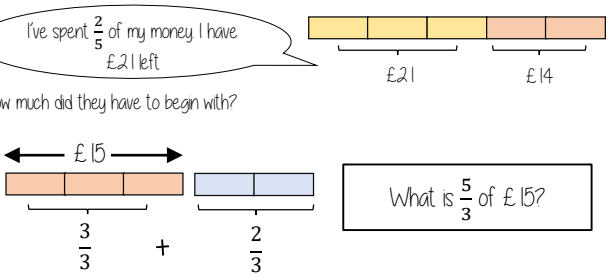
1.5 ÷ 0.05
x100 x100
150 ÷ 5 = 30

Mental methods for fractions

Use bar models where possible

I've spent $\frac{2}{5}$ of my money. I have £21 left

How much did they have to begin with?



What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

30 x 16

10 x 3 x 4 x 4
2 x 5 x 3 x 2 x 2 x 2 x 2
10 x 3 x 2 x 8
16 x 10 x 3

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

210 + 899 < 1200

This is true because even if both numbers were rounded up, they would reach 300 + 900.

The correct estimation would be 200 + 900 = 1100.

Number facts

Use 124 x 5 = 620

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

620÷ 124 = 50

For division you must consider the impact of the divisor becoming smaller or bigger.
Smaller – the answer will be bigger
(It is being shared into less parts)
Bigger – the answer will be smaller
(It is being shared into more parts)

Algebraic facts

2a + 2b = 10 — Everything x 2

0.1a + 0.1b = 0.5 — Everything ÷ 10

a + b = 5 — Add 2 to the total

a + b + 2 = 7

The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 8 — REASONING WITH NUMBER

Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Conjecture: a statement that might be true (based on some research or reasoning) but it is not proven
Counterexample: an example which shows that a particular statement is false
Expression: one or a group of numbers, variables and mathematical operations representing a number or quantity
Factor: a whole number or variable that divides exactly into another number or expression
Highest Common Factor (HCF): the largest of all the common factors shared by two (or more) numbers.
Lowest Common Multiple (LCM): the smallest whole number that is a multiple of two (or more) numbers
Multiple: the result of multiplying a number by an integer
Prime number: an integer greater than 1 which has exactly two factors – itself and 1.

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

This list continues and doesn't end

$3x$, $6x$, $9x$ ...

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2

The first prime number
The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

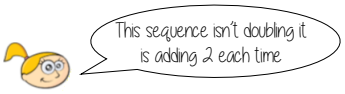
Conjectures and counterexamples

Conjecture

1, 2, 4, ...
The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

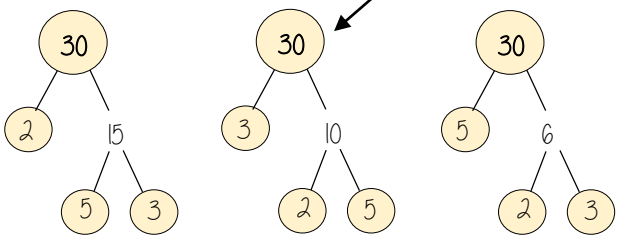
Counterexamples



Only one counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$ ← Multiplication of prime factors

Using prime factors for predictions

e.g. 60 30 x 2 2 x 3 x 5 x 2

150 30 x 5 2 x 3 x 5 x 5