YEAR 8
KNOWLEDGE ORGANISERS

BLOCK: ALGEBRAIC TECHNIQUES

Sequences

Indices

"MATHS OPENS DOORS"

YEAR 8 - ALGEBRAIC TECHNIQUES...

Sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

Keywords

Arithmetic: an arithmetic sequence is one which is made by adding the same amount each time.

Difference: the gap between two numbers, found by doing a subtraction.

Geometric: a geometric sequence is one which is made by multiplying by the same amount each time

Linear: a sequence is linear when the gap between the terms is the same each time

Non-linear: a sequence is non-linear if the gap between the terms is not the same each time.

Position: where a term is in the sequence, e.g. 10th term is the 10th number/shape along

Sequence: a set of numbers, shapes or patterns in a particular order

Term: one of the numbers, letters, shapes or patterns in a sequence, series or algebraic expression

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

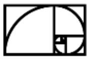
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci.

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ →

Substitute the number of the term you are looking for in place of 'n'

e.g.

1st term = 2 (1) – 5 = -3

2nd term = 2 (2) – 5 = -1

100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$3n - 4 = 201$

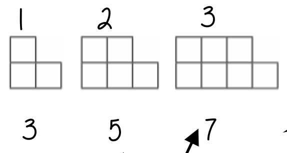
Algebraic rule

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

Sequence in a table and graphically

Position: the place in the sequence



"The **term** in **position** 3 has 7 squares"

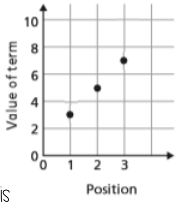
Term: the number or variable (the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

e.g.

1st term = 2 x 1² = 2

2st term = 2 x 2² = 8

100th term = 2 x 100² = 2000

$(2n)^2$

2 times n then square the answer

e.g.

1st term = (2 x 1)² = 4

2st term = (2 x 2)² = 16

100th term = (2 x 100)² = 40000

$n(n + 5)$ ←

e.g.

1st term = 1 (1 + 5) = 6

2st term = 2 (2 + 5) = 14

100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20...

$4n$

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 8 - ALGEBRAIC TECHNIQUES...

Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

Base (number): a base number is the basis of a place value number system. Successive powers of the base number are used for each column

Coefficient: a number which multiplies a variable

Exponent: another name for **index** or **power**

Index (Indices is plural): a small number written to the upper right of a number or variable which shows how many times the number or letter is multiplied by itself.

Power: another name for **index** or **exponent**

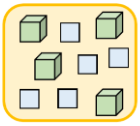
Simplify (an expression): to remove brackets, unnecessary terms and numbers, collecting like terms

Addition/ Subtraction with indices

Coefficient

Power

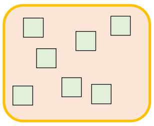
$$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}} \longrightarrow$$



Each square represents x^2 and each cube represents x^4

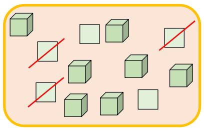
Only similar terms can be simplified. If they have different powers, they are unlike terms

$5x^2 + 2x^2 \longrightarrow$



$\longrightarrow 7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4 \longrightarrow$



$\longrightarrow 2x^2 + 7x^4$

Multiply expressions with indices

$$4b \times 3a$$
$$\equiv 4 \times b \times 3 \times a$$
$$\equiv 4 \times 3 \times b \times a$$
$$\equiv 12ab$$

$$5t \times 9t$$
$$\equiv 5 \times t \times 9 \times t$$
$$\equiv 5 \times 9 \times t \times t$$
$$\equiv 45t^2$$

$$2b^4 \times 3b^2$$
$$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$$
$$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$$
$$\equiv 6b^6$$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$$\underbrace{\begin{matrix} 3^5 \times 3^2 \\ = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) \end{matrix}} \longrightarrow 3^7$$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$$3^5 \div 3^2 \longrightarrow 3^3$$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \longrightarrow \frac{3^3}{3^0} \longrightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

Divide expressions with indices

$$\frac{24}{36} \longrightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \longrightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \longrightarrow \underbrace{\frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \longrightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms